

**Q. 1 – Q. 25 carry one mark each.**

Q.1 The possible set of eigen values of a  $4 \times 4$  skew-symmetric orthogonal real matrix is

- (A)  $\{\pm i\}$                       (B)  $\{\pm i, \pm 1\}$                       (C)  $\{\pm 1\}$                       (D)  $\{0, \pm i\}$

Q.2 The coefficient of  $(z - \pi)^2$  in the Taylor series expansion of

$$f(z) = \begin{cases} \frac{\sin z}{z - \pi} & \text{if } z \neq \pi \\ -1 & \text{if } z = \pi \end{cases}$$

around  $\pi$  is

- (A)  $\frac{1}{2}$                       (B)  $-\frac{1}{2}$                       (C)  $\frac{1}{6}$                       (D)  $-\frac{1}{6}$

Q.3 Consider  $\mathbb{R}^2$  with the usual topology. Which of the following statements are **TRUE** for all  $A, B \subseteq \mathbb{R}^2$ ?

P:  $\overline{A \cup B} = \overline{A} \cup \overline{B}$ .

Q:  $\overline{A \cap B} = \overline{A} \cap \overline{B}$ .

R:  $(A \cup B)^\circ = A^\circ \cup B^\circ$ .

S:  $(A \cap B)^\circ = A^\circ \cap B^\circ$ .

- (A) P and R only                      (B) P and S only                      (C) Q and R only                      (D) Q and S only

Q.4 Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a continuous function with  $f(1) = 5$  and  $f(3) = 11$ . If  $g(x) = \int_1^3 f(x+t)dt$  then  $g'(0)$  is equal to \_\_\_\_\_

Q.5 Let  $P$  be a  $2 \times 2$  complex matrix such that  $\text{trace}(P) = 1$  and  $\det(P) = -6$ . Then,  $\text{trace}(P^4 - P^3)$  is \_\_\_\_\_

Q.6 Suppose that  $R$  is a unique factorization domain and that  $a, b \in R$  are distinct irreducible elements. Which of the following statements is **TRUE**?

- (A) The ideal  $\langle 1 + a \rangle$  is a prime ideal  
 (B) The ideal  $\langle a + b \rangle$  is a prime ideal  
 (C) The ideal  $\langle 1 + ab \rangle$  is a prime ideal  
 (D) The ideal  $\langle a \rangle$  is not necessarily a maximal ideal

Q.7 Let  $X$  be a compact Hausdorff topological space and let  $Y$  be a topological space. Let  $f: X \rightarrow Y$  be a bijective continuous mapping. Which of the following is **TRUE**?

- (A)  $f$  is a closed map but not necessarily an open map  
 (B)  $f$  is an open map but not necessarily a closed map  
 (C)  $f$  is both an open map and a closed map  
 (D)  $f$  need not be an open map or a closed map

Q.8 Consider the linear programming problem:

$$\begin{array}{ll} \text{Maximize} & x + \frac{3}{2}y \\ \text{subject to} & 2x + 3y \leq 16, \\ & x + 4y \leq 18, \\ & x \geq 0, y \geq 0. \end{array}$$

If  $S$  denotes the set of all solutions of the above problem, then

- (A)  $S$  is empty                      (B)  $S$  is a singleton  
 (C)  $S$  is a line segment                      (D)  $S$  has positive area

Q.9 Which of the following groups has a proper subgroup that is **NOT** cyclic?

- (A)  $\mathbb{Z}_{15} \times \mathbb{Z}_{77}$   
 (B)  $S_3$   
 (C)  $(\mathbb{Z}, +)$   
 (D)  $(\mathbb{Q}, +)$

Q.10 The value of the integral

$$\int_0^\infty \int_x^\infty \left(\frac{1}{y}\right) e^{-y/2} dy dx$$

is \_\_\_\_\_

Q.11 Suppose the random variable  $U$  has uniform distribution on  $[0,1]$  and  $X = -2 \log U$ . The density of  $X$  is

(A)  $f(x) = \begin{cases} e^{-x} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$

(B)  $f(x) = \begin{cases} 2e^{-2x} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$

(C)  $f(x) = \begin{cases} \frac{1}{2} e^{-\frac{x}{2}} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$

(D)  $f(x) = \begin{cases} 1/2 & \text{if } x \in [0,2] \\ 0 & \text{otherwise} \end{cases}$

Q.12 Let  $f$  be an entire function on  $\mathbb{C}$  such that  $|f(z)| \leq 100 \log|z|$  for each  $z$  with  $|z| \geq 2$ . If  $f(i) = 2i$ , then  $f(1)$

- (A) must be 2  
 (C) must be  $i$

- (B) must be  $2i$   
 (D) cannot be determined from the given data

Q.13 The number of group homomorphisms from  $\mathbb{Z}_3$  to  $\mathbb{Z}_9$  is \_\_\_\_\_

Q.14 Let  $u(x, t)$  be the solution to the wave equation

$$\frac{\partial^2 u}{\partial x^2}(x, t) = \frac{\partial^2 u}{\partial t^2}(x, t), \quad u(x, 0) = \cos(5\pi x), \quad \frac{\partial u}{\partial t}(x, 0) = 0.$$

Then, the value of  $u(1,1)$  is \_\_\_\_\_

Q.15 Let  $f(x) = \sum_{n=1}^{\infty} \frac{\sin(nx)}{n^2}$ . Then

- (A)  $\lim_{x \rightarrow 0} f(x) = 0$   
 (C)  $\lim_{x \rightarrow 0} f(x) = \pi^2/6$

- (B)  $\lim_{x \rightarrow 0} f(x) = 1$   
 (D)  $\lim_{x \rightarrow 0} f(x)$  does not exist

- Q.16 Suppose  $X$  is a random variable with  $P(X = k) = (1 - p)^k p$  for  $k \in \{0, 1, 2, \dots\}$  and some  $p \in (0, 1)$ . For the hypothesis testing problem

$$H_0: p = \frac{1}{2} \quad H_1: p \neq \frac{1}{2}$$

consider the test “Reject  $H_0$  if  $X \leq A$  or if  $X \geq B$ ”, where  $A < B$  are given positive integers. The type-I error of this test is

- (A)  $1 + 2^{-B} - 2^{-A}$   
 (B)  $1 - 2^{-B} + 2^{-A}$   
 (C)  $1 + 2^{-B} - 2^{-A-1}$   
 (D)  $1 - 2^{-B} + 2^{-A-1}$

- Q.17 Let  $G$  be a group of order 231. The number of elements of order 11 in  $G$  is \_\_\_\_\_

- Q.18 Let  $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be defined by  $f(x, y) = (e^{x+y}, e^{x-y})$ . The area of the image of the region  $\{(x, y) \in \mathbb{R}^2: 0 < x, y < 1\}$  under the mapping  $f$  is

- (A) 1                      (B)  $e - 1$                       (C)  $e^2$                       (D)  $e^2 - 1$

- Q.19 Which of the following is a field?

- (A)  $\mathbb{C}[x]/\langle x^2 + 2 \rangle$   
 (B)  $\mathbb{Z}[x]/\langle x^2 + 2 \rangle$   
 (C)  $\mathbb{Q}[x]/\langle x^2 - 2 \rangle$   
 (D)  $\mathbb{R}[x]/\langle x^2 - 2 \rangle$

- Q.20 Let  $x_0 = 0$ . Define  $x_{n+1} = \cos x_n$  for every  $n \geq 0$ . Then

- (A)  $\{x_n\}$  is increasing and convergent  
 (B)  $\{x_n\}$  is decreasing and convergent  
 (C)  $\{x_n\}$  is convergent and  $x_{2n} < \lim_{m \rightarrow \infty} x_m < x_{2n+1}$  for every  $n \in \mathbb{N}$   
 (D)  $\{x_n\}$  is not convergent

- Q.21 Let  $C$  be the contour  $|z| = 2$  oriented in the anti-clockwise direction. The value of the integral  $\oint_C z e^{3/z} dz$  is

- (A)  $3\pi i$                       (B)  $5\pi i$                       (C)  $7\pi i$                       (D)  $9\pi i$

- Q.22 For each  $\lambda > 0$ , let  $X_\lambda$  be a random variable with exponential density  $\lambda e^{-\lambda x}$  on  $(0, \infty)$ . Then,  $\text{Var}(\log X_\lambda)$

- (A) is strictly increasing in  $\lambda$   
 (B) is strictly decreasing in  $\lambda$   
 (C) does not depend on  $\lambda$   
 (D) first increases and then decreases in  $\lambda$

- Q.23 Let  $\{a_n\}$  be the sequence of consecutive positive solutions of the equation  $\tan x = x$  and let  $\{b_n\}$  be the sequence of consecutive positive solutions of the equation  $\tan \sqrt{x} = x$ . Then
- (A)  $\sum_{n=1}^{\infty} \frac{1}{a_n}$  converges but  $\sum_{n=1}^{\infty} \frac{1}{b_n}$  diverges      (B)  $\sum_{n=1}^{\infty} \frac{1}{a_n}$  diverges but  $\sum_{n=1}^{\infty} \frac{1}{b_n}$  converges
- (C) Both  $\sum_{n=1}^{\infty} \frac{1}{a_n}$  and  $\sum_{n=1}^{\infty} \frac{1}{b_n}$  converge      (D) Both  $\sum_{n=1}^{\infty} \frac{1}{a_n}$  and  $\sum_{n=1}^{\infty} \frac{1}{b_n}$  diverge
- Q.24 Let  $f$  be an analytic function on  $\bar{D} = \{z \in \mathbb{C} : |z| \leq 1\}$ . Assume that  $|f(z)| \leq 1$  for each  $z \in \bar{D}$ . Then, which of the following is **NOT** a possible value of  $(e^f)''(0)$ ?
- (A) 2                                      (B) 6                                      (C)  $\frac{7}{9}e^{1/9}$                                       (D)  $\sqrt{2} + i\sqrt{2}$
- Q.25 The number of non-isomorphic abelian groups of order 24 is \_\_\_\_\_

**Q. 26 to Q. 55 carry two marks each.**

- Q.26 Let  $V$  be the real vector space of all polynomials in one variable with real coefficients and having degree at most 20. Define the subspaces

$$W_1 = \left\{ p \in V : p(1) = 0, \quad p\left(\frac{1}{2}\right) = 0, \quad p(5) = 0, \quad p(7) = 0 \right\},$$

$$W_2 = \left\{ p \in V : p\left(\frac{1}{2}\right) = 0, \quad p(3) = 0, \quad p(4) = 0, \quad p(7) = 0 \right\}.$$

Then the dimension of  $W_1 \cap W_2$  is \_\_\_\_\_

- Q.27 Let  $f, g : [0,1] \rightarrow \mathbb{R}$  be defined by

$$f(x) = \begin{cases} x & \text{if } x = \frac{1}{n} \text{ for } n \in \mathbb{N} \\ 0 & \text{otherwise} \end{cases}$$

and

$$g(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \cap [0,1] \\ 0 & \text{otherwise.} \end{cases}$$

Then

- (A) Both  $f$  and  $g$  are Riemann integrable  
 (B)  $f$  is Riemann integrable and  $g$  is Lebesgue integrable  
 (C)  $g$  is Riemann integrable and  $f$  is Lebesgue integrable  
 (D) Neither  $f$  nor  $g$  is Riemann integrable
- Q.28 Consider the following linear programming problem:

$$\begin{array}{ll} \text{Maximize} & x + 3y + 6z - w \\ \text{subject to} & 5x + y + 6z + 7w \leq 20, \\ & 6x + 2y + 2z + 9w \leq 40, \\ & x \geq 0, \quad y \geq 0, \quad z \geq 0, \quad w \geq 0. \end{array}$$

Then the optimal value is \_\_\_\_\_

- Q.29 Suppose  $X$  is a real-valued random variable. Which of the following values **CANNOT** be attained by  $E[X]$  and  $E[X^2]$ , respectively?
- (A) 0 and 1                                      (B) 2 and 3                                      (C)  $\frac{1}{2}$  and  $\frac{1}{3}$                                       (D) 2 and 5

Q.30 Which of the following subsets of  $\mathbb{R}^2$  is **NOT** compact?

- (A)  $\{(x, y) \in \mathbb{R}^2 : -1 \leq x \leq 1, y = \sin x\}$   
 (B)  $\{(x, y) \in \mathbb{R}^2 : -1 \leq y \leq 1, y = x^8 - x^3 - 1\}$   
 (C)  $\{(x, y) \in \mathbb{R}^2 : y = 0, \sin(e^{-x}) = 0\}$   
 (D)  $\{(x, y) \in \mathbb{R}^2 : x > 0, y = \sin\left(\frac{1}{x}\right)\} \cap \{(x, y) \in \mathbb{R}^2 : x > 0, y = \frac{1}{x}\}$

Q.31 Let  $M$  be the real vector space of  $2 \times 3$  matrices with real entries. Let  $T: M \rightarrow M$  be defined by

$$T \left( \begin{bmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \end{bmatrix} \right) = \begin{bmatrix} -x_6 & x_4 & x_1 \\ x_3 & x_5 & x_2 \end{bmatrix}.$$

The determinant of  $T$  is \_\_\_\_\_

Q.32 Let  $\mathcal{H}$  be a Hilbert space and let  $\{e_n : n \geq 1\}$  be an orthonormal basis of  $\mathcal{H}$ . Suppose  $T: \mathcal{H} \rightarrow \mathcal{H}$  is a bounded linear operator. Which of the following **CANNOT** be true?

- (A)  $T(e_n) = e_1$  for all  $n \geq 1$   
 (B)  $T(e_n) = e_{n+1}$  for all  $n \geq 1$   
 (C)  $T(e_n) = \sqrt{\frac{n+1}{n}} e_n$  for all  $n \geq 1$   
 (D)  $T(e_n) = e_{n-1}$  for all  $n \geq 2$  and  $T(e_1) = 0$

Q.33 The value of the limit

$$\lim_{n \rightarrow \infty} \frac{2^{-n^2}}{\sum_{k=n+1}^{\infty} 2^{-k^2}}$$

is

- (A) 0                      (B) some  $c \in (0, 1)$       (C) 1                      (D)  $\infty$

Q.34 Let  $f: \mathbb{C} \setminus \{3i\} \rightarrow \mathbb{C}$  be defined by  $f(z) = \frac{z-i}{iz+3}$ . Which of the following statements about  $f$  is **FALSE**?

- (A)  $f$  is conformal on  $\mathbb{C} \setminus \{3i\}$   
 (B)  $f$  maps circles in  $\mathbb{C} \setminus \{3i\}$  onto circles in  $\mathbb{C}$   
 (C) All the fixed points of  $f$  are in the region  $\{z \in \mathbb{C} : \text{Im}(z) > 0\}$   
 (D) There is no straight line in  $\mathbb{C} \setminus \{3i\}$  which is mapped onto a straight line in  $\mathbb{C}$  by  $f$

Q.35 The matrix  $A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{bmatrix}$  can be decomposed uniquely into the product  $A = LU$ , where

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \text{ and } U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}. \text{ The solution of the system } LX = [1 \ 2 \ 2]^t \text{ is}$$

- (A)  $[1 \ 1 \ 1]^t$       (B)  $[1 \ 1 \ 0]^t$       (C)  $[0 \ 1 \ 1]^t$       (D)  $[1 \ 0 \ 1]^t$

- Q.36 Let  $S = \{x \in \mathbb{R} : x \geq 0, \sum_{n=1}^{\infty} x^{\sqrt{n}} < \infty\}$ . Then the supremum of  $S$  is  
 (A) 1 (B)  $\frac{1}{e}$  (C) 0 (D)  $\infty$
- Q.37 The image of the region  $\{z \in \mathbb{C} : \operatorname{Re}(z) > \operatorname{Im}(z) > 0\}$  under the mapping  $z \mapsto e^{z^2}$  is  
 (A)  $\{w \in \mathbb{C} : \operatorname{Re}(w) > 0, \operatorname{Im}(w) > 0\}$  (B)  $\{w \in \mathbb{C} : \operatorname{Re}(w) > 0, \operatorname{Im}(w) > 0, |w| > 1\}$   
 (C)  $\{w \in \mathbb{C} : |w| > 1\}$  (D)  $\{w \in \mathbb{C} : \operatorname{Im}(w) > 0, |w| > 1\}$
- Q.38 Which of the following groups contains a unique normal subgroup of order four?  
 (A)  $\mathbb{Z}_2 \oplus \mathbb{Z}_4$  (B) The dihedral group,  $D_4$ , of order eight  
 (C) The quaternion group,  $Q_8$  (D)  $\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$
- Q.39 Let  $B$  be a real symmetric positive-definite  $n \times n$  matrix. Consider the inner product on  $\mathbb{R}^n$  defined by  $\langle X, Y \rangle = Y^t B X$ . Let  $A$  be an  $n \times n$  real matrix and let  $T: \mathbb{R}^n \rightarrow \mathbb{R}^n$  be the linear operator defined by  $T(X) = AX$  for all  $X \in \mathbb{R}^n$ . If  $S$  is the adjoint of  $T$ , then  $S(X) = CX$  for all  $X \in \mathbb{R}^n$ , where  $C$  is the matrix  
 (A)  $B^{-1} A^t B$  (B)  $BA^t B^{-1}$  (C)  $B^{-1} AB$  (D)  $A^t$
- Q.40 Let  $X$  be an arbitrary random variable that takes values in  $\{0, 1, \dots, 10\}$ . The minimum and maximum possible values of the variance of  $X$  are  
 (A) 0 and 30 (B) 1 and 30 (C) 0 and 25 (D) 1 and 25
- Q.41 Let  $M$  be the space of all  $4 \times 3$  matrices with entries in the finite field of three elements. Then the number of matrices of rank three in  $M$  is  
 (A)  $(3^4 - 3)(3^4 - 3^2)(3^4 - 3^3)$   
 (B)  $(3^4 - 1)(3^4 - 2)(3^4 - 3)$   
 (C)  $(3^4 - 1)(3^4 - 3)(3^4 - 3^2)$   
 (D)  $3^4(3^4 - 1)(3^4 - 2)$
- Q.42 Let  $V$  be a vector space of dimension  $m \geq 2$ . Let  $T: V \rightarrow V$  be a linear transformation such that  $T^{n+1} = 0$  and  $T^n \neq 0$  for some  $n \geq 1$ . Then which of the following is necessarily **TRUE**?  
 (A)  $\operatorname{Rank}(T^n) \leq \operatorname{Nullity}(T^n)$  (B)  $\operatorname{trace}(T) \neq 0$   
 (C)  $T$  is diagonalizable (D)  $n = m$
- Q.43 Let  $X$  be a convex region in the plane bounded by straight lines. Let  $X$  have 7 vertices. Suppose  $f(x, y) = ax + by + c$  has maximum value  $M$  and minimum value  $N$  on  $X$  and  $N < M$ . Let  $S = \{P : P \text{ is a vertex of } X \text{ and } N < f(P) < M\}$ . If  $S$  has  $n$  elements, then which of the following statements is **TRUE**?  
 (A)  $n$  cannot be 5 (B)  $n$  can be 2  
 (C)  $n$  cannot be 3 (D)  $n$  can be 4
- Q.44 Which of the following statements are **TRUE**?  
 P: If  $f \in L^1(\mathbb{R})$ , then  $f$  is continuous.  
 Q: If  $f \in L^1(\mathbb{R})$  and  $\lim_{|x| \rightarrow \infty} f(x)$  exists, then the limit is zero.  
 R: If  $f \in L^1(\mathbb{R})$ , then  $f$  is bounded.  
 S: If  $f \in L^1(\mathbb{R})$  is uniformly continuous, then  $\lim_{|x| \rightarrow \infty} f(x)$  exists and equals zero.  
 (A) Q and S only (B) P and R only (C) P and Q only (D) R and S only

Q.45 Let  $u$  be a real valued harmonic function on  $\mathbb{C}$ . Let  $g: \mathbb{R}^2 \rightarrow \mathbb{R}$  be defined by

$$g(x, y) = \int_0^{2\pi} u(e^{i\theta}(x + iy)) \sin \theta \, d\theta.$$

Which of the following statements is **TRUE**?

- (A)  $g$  is a harmonic polynomial
- (B)  $g$  is a polynomial but not harmonic
- (C)  $g$  is harmonic but not a polynomial
- (D)  $g$  is neither harmonic nor a polynomial

Q.46 Let  $S = \{z \in \mathbb{C} : |z| = 1\}$  with the induced topology from  $\mathbb{C}$  and let  $f: [0, 2] \rightarrow S$  be defined as  $f(t) = e^{2\pi it}$ . Then, which of the following is **TRUE**?

- (A)  $K$  is closed in  $[0, 2] \Rightarrow f(K)$  is closed in  $S$
- (B)  $U$  is open in  $[0, 2] \Rightarrow f(U)$  is open in  $S$
- (C)  $f(X)$  is closed in  $S \Rightarrow X$  is closed in  $[0, 2]$
- (D)  $f(Y)$  is open in  $S \Rightarrow Y$  is open in  $[0, 2]$

Q.47 Assume that all the zeros of the polynomial  $a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$  have negative real parts. If  $u(t)$  is any solution to the ordinary differential equation

$$a_n \frac{d^n u}{dt^n} + a_{n-1} \frac{d^{n-1} u}{dt^{n-1}} + \dots + a_1 \frac{du}{dt} + a_0 u = 0,$$

then  $\lim_{t \rightarrow \infty} u(t)$  is equal to

- (A) 0
- (B) 1
- (C)  $n - 1$
- (D)  $\infty$

## Common Data Questions

### Common Data for Questions 48 and 49:

Let  $c_{00}$  be the vector space of all complex sequences having finitely many non-zero terms. Equip  $c_{00}$  with the inner product  $\langle x, y \rangle = \sum_{n=1}^{\infty} x_n \overline{y_n}$  for all  $x = (x_n)$  and  $y = (y_n)$  in  $c_{00}$ . Define  $f: c_{00} \rightarrow \mathbb{C}$  by  $f(x) = \sum_{n=1}^{\infty} \frac{x_n}{n}$ . Let  $N$  be the kernel of  $f$ .

Q.48 Which of the following is **FALSE**?

- (A)  $f$  is a continuous linear functional
- (B)  $\|f\| \leq \frac{\pi}{\sqrt{6}}$
- (C) There does not exist any  $y \in c_{00}$  such that  $f(x) = \langle x, y \rangle$  for all  $x \in c_{00}$
- (D)  $N^\perp \neq \{0\}$

Q.49 Which of the following is **FALSE**?

- (A)  $c_{00} \neq N$
- (B)  $N$  is closed
- (C)  $c_{00}$  is not a complete inner product space
- (D)  $c_{00} = N \oplus N^\perp$

**Common Data for Questions 50 and 51:**

Let  $X_1, X_2, \dots, X_n$  be an i.i.d. random sample from exponential distribution with mean  $\mu$ . In other words, they have density

$$f(x) = \begin{cases} \frac{1}{\mu} e^{-x/\mu} & \text{if } x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Q.50 Which of the following is **NOT** an unbiased estimate of  $\mu$ ?

- (A)  $X_1$   
 (B)  $\frac{1}{n-1}(X_2 + X_3 + \dots + X_n)$   
 (C)  $n \cdot (\min\{X_1, X_2, \dots, X_n\})$   
 (D)  $\frac{1}{n} \max\{X_1, X_2, \dots, X_n\}$

Q.51 Consider the problem of estimating  $\mu$ . The m.s.e (mean square error) of the estimate

$$T(X) = \frac{X_1 + X_2 + \dots + X_n}{n + 1}$$

is

- (A)  $\mu^2$                       (B)  $\frac{1}{n+1}\mu^2$                       (C)  $\frac{1}{(n+1)^2}\mu^2$                       (D)  $\frac{n^2}{(n+1)^2}\mu^2$

**Linked Answer Questions****Statement for Linked Answer Questions 52 and 53:**

Let  $X = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\} \cup ([-1, 1] \times \{0\}) \cup (\{0\} \times [-1, 1])$ .

Let  $n_0 = \max\{k : k < \infty, \text{ there are } k \text{ distinct points } p_1, \dots, p_k \in X \text{ such that } X \setminus \{p_1, \dots, p_k\} \text{ is connected}\}$

Q.52 The value of  $n_0$  is \_\_\_\_\_

Q.53 Let  $q_1, \dots, q_{n_0+1}$  be  $n_0 + 1$  distinct points and  $Y = X \setminus \{q_1, \dots, q_{n_0+1}\}$ . Let  $m$  be the number of connected components of  $Y$ . The maximum possible value of  $m$  is \_\_\_\_\_

**Statement for Linked Answer Questions 54 and 55:**

Let  $W(y_1, y_2)$  be the Wronskian of two linearly independent solutions  $y_1$  and  $y_2$  of the equation  $y'' + P(x)y' + Q(x)y = 0$ .

Q.54 The product  $W(y_1, y_2)P(x)$  equals

- (A)  $y_2 y_1'' - y_1 y_2''$                       (B)  $y_1 y_2' - y_2 y_1'$   
 (C)  $y_1' y_2'' - y_2' y_1''$                       (D)  $y_2' y_1' - y_1'' y_2''$

Q.55 If  $y_1 = e^{2x}$  and  $y_2 = xe^{2x}$ , then the value of  $P(0)$  is

- (A) 4                      (B) -4                      (C) 2                      (D) -2



**General Aptitude (GA) Questions****Q. 56 – Q. 60 carry one mark each.**

Q.56 A number is as much greater than 75 as it is smaller than 117. The number is:  
(A) 91 (B) 93 (C) 89 (D) 96

Q.57 The professor ordered to the students to go out of the class.  
I II III IV

Which of the above underlined parts of the sentence is grammatically incorrect?

(A) I (B) II (C) III (D) IV

Q.58 Which of the following options is the closest in meaning to the word given below:

Primeval

(A) Modern (B) Historic  
(C) Primitive (D) Antique

Q.59 Friendship, no matter how \_\_\_\_\_ it is, has its limitations.

(A) cordial  
(B) intimate  
(C) secret  
(D) pleasant

Q.60 Select the pair that best expresses a relationship similar to that expressed in the pair:  
**Medicine: Health**

(A) Science: Experiment (B) Wealth: Peace  
(C) Education: Knowledge (D) Money: Happiness

**Q. 61 to Q. 65 carry two marks each.**

Q.61 X and Y are two positive real numbers such that  $2X + Y \leq 6$  and  $X + 2Y \leq 8$ . For which of the following values of  $(X, Y)$  the function  $f(X, Y) = 3X + 6Y$  will give maximum value?

(A)  $(4/3, 10/3)$   
(B)  $(8/3, 20/3)$   
(C)  $(8/3, 10/3)$   
(D)  $(4/3, 20/3)$

Q.62 If  $|4X - 7| = 5$  then the values of  $2|X| - |-X|$  is:

(A) 2, 1/3 (B) 1/2, 3 (C) 3/2, 9 (D) 2/3, 9

- Q.63 Following table provides figures (in rupees) on annual expenditure of a firm for two years - 2010 and 2011.

Category	2010	2011
Raw material	5200	6240
Power & fuel	7000	9450
Salary & wages	9000	12600
Plant & machinery	20000	25000
Advertising	15000	19500
Research & Development	22000	26400

In 2011, which of the following two categories have registered increase by same percentage?

- (A) Raw material and Salary & wages  
(B) Salary & wages and Advertising  
(C) Power & fuel and Advertising  
(D) Raw material and Research & Development
- Q.64 A firm is selling its product at Rs. 60 per unit. The total cost of production is Rs. 100 and firm is earning total profit of Rs. 500. Later, the total cost increased by 30%. By what percentage the price should be increased to maintained the same profit level.
- (A) 5                      (B) 10                      (C) 15                      (D) 30
- Q.65 Abhishek is elder to Savar.  
Savar is younger to Anshul.

Which of the given conclusions is logically valid and is inferred from the above statements?

- (A) Abhishek is elder to Anshul  
(B) Anshul is elder to Abhishek  
(C) Abhishek and Anshul are of the same age  
(D) No conclusion follows

**END OF THE QUESTION PAPER**